Analysis of fluid induced vibration of cryogenic pipes in consideration of the cooling effect [†]

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Abstract

The purpose of system analysis using fluid induced vibration is to identify the problems of the system in advance by analyzing the vibration behavior of the system excited by fluid flow. Fluid-induced vibration analysis methods, developed so far, generally use the numerical analysis method to analyze the fluid flowing inside the pipe and the infinitesimal elements at normal temperature on the basis of the governing equation obtained by applying Newton's Second Law and the momentum equation. However, as the fluid temperature changes greatly at low temperature, fluid-induced vibration analysis methods for normal temperature cannot be applied. This study investigated methods of analyzing fluid-induced vibration in consideration of the cooling effect. In consideration of the changes in the properties of the fluid and system relative to temperature, vibration behavior was analyzed numerically by means of the equation of motion. As a result, the natural frequency of the system tends to change because of the changes of the properties of materials even when the flux is constant inside the pipe, and the vibration behavior of the system was compared to that in case of normal temperature to analyze how much influence the cooling effect has on the vibration behavior of the system.

Keywords: Fluid induced vibration; Pipe dynamics; Natural frequency; Cooling effect

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1. Introduction

There may be several causes of the vibration occurring in a pipeline system transporting fluid: e.g., in case the vibration of the machine caused by the operation of a motor or pump is transferred to the pipeline, and external excitation forces, such as an earthquake, are transmitted to the pipe system. In these cases the vibration behavior is numerically analyzed by applying an appropriate law of motion to the infinitesimal elements of the fluid and pipe. If vibration is caused simply by the flux and pressure of the fluid when there is no external excitation force, it is called fluid induced vibration. If the fluid flows at a high velocity, or if the internal pressure changes greatly, the dynamic properties of the pipe may be affected, and fluid induced vibration may be generated. The relationship between the structural properties of the pipe and the physical properties of the fluid must be taken into consideration before this pipe system is analyzed.

Fluid-induced vibration was first studied by Ashley and Haviland. They studied the transverse vibration of a pipe inside which fluid flows. Afterwards, Housner proposed a modified equation of motion which is obtained by applying Coriolis force to the previously derived equation of motion. However, as these equations of motion were derived by analyzing the properties of the fluid flow first, and applying them to the dynamic equation of the pipe, the fluid-induced vibration behavior of the pipe could not be correctly reflected. To analyze the fluid induced vibration of the pipe in relation to the abnormal flow of the fluid

flowing inside the pipe, Lee [1] studied the equation of motion in consideration of both the dynamic properties of the pipe and the physical properties of the fluid. The advantage of this equation is that it enables a more accurate analysis than individually derived equations, but its disadvantage is the difficulty of analysis.

Examples of those methods of analyzing governing equations which couple the properties of the fluid and the dynamic properties of the pipe are the finite element method, Galerkin's method, Fourier and Laplace transformation, the spectral element analysis method, etc. Most of these methods transform the governing equation into a form that can be numerically calculated and obtain approximate solutions. The differences between these methods are focused mainly on the reduction of analysis time and the accuracy of analysis. However, most methods of analyzing fluid-induced vibration are limited to analysis of the normal temperature flow of the fluid. Accordingly, in cases where there are obvious changes in the temperature of the internal fluid in a cryogenic system, it is impossible to use the aforementioned methods for analysis. The reason is the temperature change of the internal fluid is accompanied by heat transfer to and from the pipe, and accordingly, the properties of the flow and the physical properties of the pipe change at the same time. In this case the temperature of the pipe declines owing to the temperature of the fluid, whereas the temperature of the fluid rises, but as fluid of a different temperature flows into the control volume, analysis is not easy. In addition, as the fluid flows, the temperature distribution inside the pipe is not consistent. Therefore, it must be taken into consideration for analysis. However, no paper relating to such analysis or research has ever been published so far.

This study analyzed the fluid-induced vibration of the cryogenic pipe in consideration of the cooling effect. To this end, existing dynamic equations of motion were used, and various methods of taking the cooling effect into consideration were proposed. In addition, a modified equation of motion was used to analyze the stability of the pipeline, and lastly the responses to the transverse vibration of the pipeline were simulated for comparison with the result at the normal temperature so that the influence of the cooling effect on the vibration behavior of the pipeline was quantitatively analyzed.

2. System analysis

2.1 Governing equations

To derive the equation of motion of the system related to the fluid flow and the structure of the pipe, the system as illustrated in Fig. 1 is assumed. In the initial state the system forms an angle of α with the horizontal line as shown in (a) of Fig. 1, and maintains the form of a straight line. When the fluid flows inside the pipe, the pipe is assumed to move $\boldsymbol{u}$ in the x direction and w in the y direction. The free body diagram of the pipeline and fluid particles in relation to infinitesimal elements can be expressed as illustrated in (b) and (c) of Fig. 1. Here, T is the tension of the pipeline, M bending moment, Q shear force, Ndx vertical force, $\tau S dx$ the shear force attributed to the viscosity of the fluid particle, c the flux, and P the pressure of the fluid. The force, pressure and deformation properties such as velocity, angle shown in the free body diagram of the pipeline and fluid particles can be expressed by means of the Taylor series. If the size of the lateral and y direction displacement of the pipeline generated by the flow of the fluid is assumed to be small, the shape of the flow can be approximated to one-dimensional, abnormal and viscous flow, and if the system is assumed to be 2-dimensional, the governing equation of $\boldsymbol{x}$ and $\boldsymbol{y}$ with regard to the infinitesimal elements of the pipeline can be expressed by means of Newton's Second Law as shown below [1].

$$T' - Q'w' - Qw'' + Nw' + \tau S - m_p g_x = m_p \ddot{u} \tag{1}$$

$$Q' + Tw'' + T'w' - N + \tau S w' - m_p g_y = m_p \ddot{w} \tag{2}$$

Here, $\boldsymbol{m_p}$ is the mass per unit length of the pipeline, and $\boldsymbol{g_x}$ and $\boldsymbol{g_y}$ the gravity acceleration of $\boldsymbol{x}$ direction and $\boldsymbol{y}$ direction respectively. $\boldsymbol{\tau}$ is the shear stress between the pipeline and the fluid particle, and S the contact area per unit length of the pipeline.
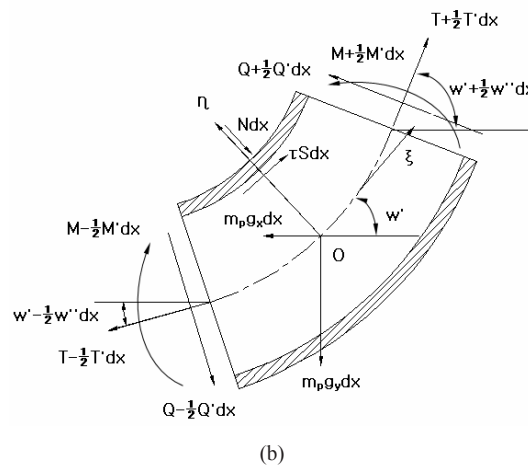
The continuity equation and momentum equation can be applied to fluid particles by means of the following equation [1].

$$\frac{d}{dt} \int_{cv} \rho_w dV + \int_{cs} \rho_w{}^{CS}V \cdot dA = 0 \tag{3}$$
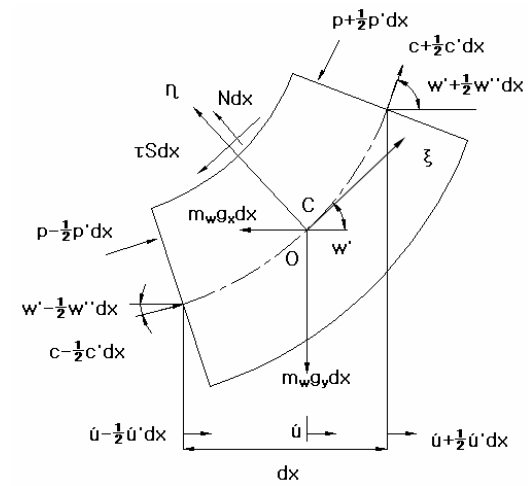
$$\sum F = \frac{d}{dt} \int_{CS} {}^S V \rho_w dV + \int_{CS} {}^S V \rho_w{}^{CS}V \cdot dA + \int_{CV} ({}^R a^C + 2\,{}^R \omega^S \times {}^S V_0) \rho_w dV \tag{4}$$
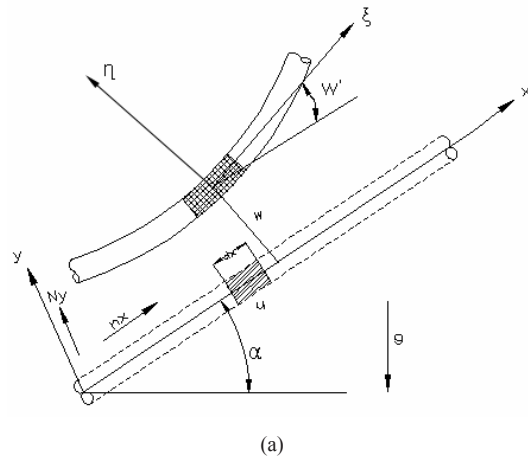
(a)

(b)

(c)

Fig. 1. A usual pipeline element (a) , free body diagram of the infinitesimal pipe element (b) and the fluid element (c).

Here, ρ_w is the density of the fluid, dV the control volume of the fluid, and dA the vector which is perpendicular to the area and whose size is the area. The velocity and acceleration included in the above equation can be expressed by means of the following equation [2].

$$^{R}a^{C} = \ddot{u}n_{x} + \ddot{w}n_{y}$$

$$^{R}w^{S} = \dot{w}'n_{z}$$

$$^{S}V_{0} = (c \mp \tfrac{1}{2}c'dx \mp \tfrac{1}{2}\dot{u}dx)n_{x} + (cw' \mp \tfrac{1}{2}c'w'dx \mp \tfrac{1}{2}cw'')n_{y}$$

$$^{CS}V = (c \mp \tfrac{1}{2}c'dx)n_{x} + (cw' \mp \tfrac{1}{2}c'w'dx \mp \tfrac{1}{2}cw'')n_{y} \quad (5)$$

If Eq. (5) is substituted in Eqs. (3) and (4), and Eqs. (1) and (2) are used for simplification, the following equation will be obtained [3].

$$\frac{\hat{\rho}_{w}}{\rho_{w}} + \frac{\hat{A}}{A} + c' = 0 \tag{6}$$

$$(pA)' + Nw' + \tau S + m_{w}(g_{x} + \ddot{u} + \dot{c} + cc' + c\dot{u}') = 0 \tag{7}$$

$$(pAw')' - N + \tau Sw' = -m_{w}(g_{r} + \ddot{w} + \dot{c}w' + 2c\dot{w}' \\ + c^{2}w'' + cc'w') \tag{8}$$

Here, the hat symbol ($\wedge$) refers to material derivative, and A refers to the cross sectional area in relation to the control volume of the fluid. Shear force Q and tension T can be expressed as derivatives of displacement by means of the elastic theory as follows [4]:

$$Q = -EI_{p}w'''' \tag{9}$$

$$T = EA_{p}u' \tag{10}$$

Here, E is the modulus of elasticity of the pipeline, I_p the second moment of the area with respect to the cross section of the pipeline, and A_p the cross sectional area of the pipeline. To transform the expression of the continuity equation of Eq. (6), the definition of the bulk modulus (E_v) can be used as shown below [5]:

$$\frac{\hat{\rho}_{w}}{\rho_{w}} = \frac{\hat{p}}{E_{v}} \tag{11}$$

The second term of the continuity equation can be expressed by means of the membrane stress theory and the expressions of the material derivative as follows:

$$\frac{\hat{A}}{A} = \frac{D}{Et}\hat{p} \qquad (12)$$

D is the inside diameter of the pipe, while t is the thickness of the pipe. A total of 5 governing equations are derived: 2 obtained by applying Newton's Second Law to the pipe, and 3 continuity equations and momentum equations for the fluid. However, the above system has 4 variables in total: x, y displacement of the pipe, and the velocity and pressure of the fluid. Accordingly, the remaining equations are not independent. Consequently, the above equation can be used to eliminate the remaining the 4 equations.

Primarily, if Eqs. (9) and (10) are substituted in Eqs. (1) and (2), and Eqs. (7) and (8) are used to substitute the equation derived with τs in Eq. (1) for simplification, the following modified equation can be obtained.

$$m\ddot{u} = EA_p u'' - (pA)' - m_w(\dot{c} + cc') - mg_x \qquad (13)$$

Here, m is the sum of m_p and m_w. Normal force N can be eliminated from Eq. (2) in the same way, and this equation can be modified as follows:

$$m\ddot{w} + EI_p w'''' = -(pAw')' - mg_y - m_w(\dot{c}w' + 2c\dot{w}' + c^2 w'' + cc'w') \qquad (14)$$

If Eqs. (11) and (12) are substituted in Eq. (6) for simplification, a modified continuity equation can be expressed as follows:

$$(\hat{p}A) + m_w a^2 c' = 0 \qquad (15)$$

Here, a is the pressure wave speed and can be expressed as follows:

$$a^2 = \frac{E_v / \rho_w}{1 + E_v D / Et} \qquad (16)$$

If Eq. (7) is multiplied by w' to combine momentum Eqs. (7) and (8) for the fluid, and one equation is subtracted from the other, and the high-order terms and non-linear terms are ignored for simplification, it can be expressed as follows:

$$(pA)' + \tau S + m_w(g_x + g_y w' + \dot{c} + cc') = 0 \qquad (17)$$

If the coefficient of friction of the Darch-Weisbach equation is used to express the shear stress term in Eq. (17) differently, it will be as follows [1]:

$$\tau = \frac{1}{4} f \frac{\rho_w}{2} c^2 \qquad (18)$$

$$\tau S = \frac{1}{4}\pi D^2 f \frac{\rho_w}{2D} c^2 = Af \frac{\rho_w}{2D} c^2 \qquad (19)$$

S is the contact area per unit length of the pipeline, and f the coefficient of friction of the pipeline. If Eq. (19) is substituted in Eq. (17) for simplification, the equation of motion of the fluid can be expressed as follows:

$$(pA)' + m_w f \frac{c^2}{2D} + m_w(g_x + g_y w' + \dot{c} + cc') = 0 \quad (20)$$

Therefore, if the 4 equations derived here are used, the fluid-induced vibration of the pipeline at the normal temperature can be analyzed.

2.2 Weak formulations

To analyze the fluid-induced vibration of the pipeline by using the 4 equations derived above, the above equation must be converted into a weak formulation. Assuming that the pipe is installed horizontally, the x direction of gravity can be ignored. Assuming that initial tension T_0 is applied to both ends of the pipe, after Eq. (20) is inserted in Eq. (13) for simplification, Eq. (13) can be converted as follows [3]:

$$m\ddot{u} - (EA_p - T_0)u'' = m_w g w' + m_w \frac{f}{2D} c^2 \qquad (21)$$

To analyze the abnormal state, the pressure and velocity of the fluid can be expressed as follows with the value of the steady state and the value of the perturbation component [6]:

$$\begin{aligned} c &= c_0 + c_d \\ p &= p_0 + p_d \end{aligned} \qquad (22)$$

Here, C_o, P_o is the value of the normal state of the velocity and pressure, whereas C_d, P_d is the perturbation component of the velocity and pressure. If the above values are substituted in Eq. (21), it will be expressed as follows:

$$m\ddot{u} - (EA_p - T_0)u'' = m_w gw' + m_w \frac{f}{2D}c_0^2 + m_w \frac{f}{D}c_0 c_d$$
(23)

If Eqs. (14), (15) and (20) are expressed in a similar way, they can be expressed as follows:

$$m\ddot{w} + EI_p w'''' + (p_0 A - T_0)w'' + m_w c_0^2 w'' + 2m_w c_0 \dot{w}' = 0$$
(24)

$$\dot{p}_d A + m_w a^2 c_d' = 0$$
(25)

$$p_d' + \rho_w \dot{c}_d = -\rho_w gw' - \rho_w \frac{f}{2D}c_0^2 - \rho_w \frac{f}{D}c_0 c_d - \rho_w c_0 c_d'$$
(26)

The weak formulation of the above governing equation can be expressed as follows by multiplying both sides of the equation by the variation and thus integrating the entire section [7].

$$\int_0^L \delta u m\ddot{u}\,dx - (EA_p - T_0)\int_0^L \delta u u''\,dx =$$

$$m_w g \int_0^L \delta u w'\,dx + m_w \frac{f}{2D}c_0^2 \int_0^L \delta u\,dx + m_w \frac{f}{D}c_0 \int_0^L \delta u c_d\,dx$$

$$\int_0^L \delta w m\ddot{w}\,dx + EI_p \int_0^L \delta w w''''\,dx + (p_0 A - T_0 + m_w c_0^2)\int_0^L \delta w w''\,dx$$

$$+\int_0^L \delta w 2m_w c_0 \dot{w}'\,dx = 0$$

$$\int_0^L \delta p_d \dot{p}_d\,dx + \rho_w a^2 \int_0^L \delta p_d c_d'\,dx = 0$$

$$\int_0^L \delta c_d p_d'\,dx + \rho_w \int_0^L \delta c_d \dot{c}_d\,dx = -\rho_w g \int_0^L \delta c_d w'\,dx -$$

$$\rho_w \frac{f}{2D}c_0^2 \int_0^L \delta c_d\,dx - \rho_w \frac{f}{D}c_0 \int_0^L \delta c_d c_d\,dx - \rho_w c_0 \int_0^L \delta c_d c_d'\,dx$$
(27)

If the above equations are multiplied by an appropriate mode function for integration and simplification, it can be expressed as the following matrix equation [8]:

$$M_u \ddot{U} + K_u U = F_u$$
(28)

$$M_w \ddot{w} + C_w \dot{w} + K_w w = 0$$
(29)

$$C_p \dot{P}_d = F_p$$
(30)

$$C_c \dot{C}_d = F_c$$
(31)

Accordingly, if Eqs. (28)~(31) are calculated under appropriate initial conditions, the responses on the lateral and y direction of the pipeline can be calculated in relation to time.

2.3 Consideration of the cooling effect

In an actual cryogenic system the fluid initially starts to flow inside the pipe at a normal temperature, and the temperature decreases considerably with time. The pipeline used in this case has two layers with vacuum in-between so that heat from the outside is blocked. If the effect of the radiation ignored, the decline of the temperature of the pipeline can be thought to be caused by the heat transfer with the low temperature of the fluid. It is extremely difficult to develop a method of considering and analyzing the change in the temperature of the fluid in the pipeline over time, and apply it. To do so, the temperature of the fluid in the pipeline over time, the contact surface of the pipe, the temperature gradient in relation to the outside and the heat transfer effect occurring inside the fluid must be taken into consideration. Accordingly, this study attempts to use the lumped system method to analyze it. As this method assumes that the overall temperature of the pipeline is uniform, it is easy to analyze the change in temperature over time, but pre-analysis is required for this assumption to be valid. Thus, the Biot Number was calculated and the validity of this assumption was tested for each time.

The system is assumed as follows for analysis. As illustrated in Fig. 2, the system is assumed to be a straight double-wall pipeline, and the two ends of the pipeline are simply supported. The inflowing fluid is helium, initially at a temperature of 200K, and the temperature of the pipeline at this time is assumed to be 300K. The length of the pipeline is assumed to be 5 m, its inside diameter 18.6 mm, its thickness 3.4 mm, and its mass per unit length 1.946 kg/m.
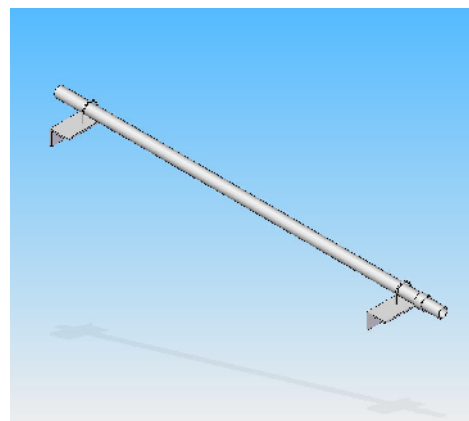

Fig. 2. A usually horizontal pipeline which is simply supported.

If the cryogenic helium flows into the pipeline, the temperature of the helium and that of the pipeline change over time due to the effect of the heat transfer between the two. The actual temperature change varies depending on the length and thickness of the pipe, and the temperature distribution of the helium is not the same at all points, and varies over time. However, as the analysis in this case is very complicated, a lumped system analysis will be conducted here. If the cryogenic pipe is assumed to be a lumped system, its temperature will decrease through heat exchange with the helium. If the temperature of the inflowing helium is assumed to be consistent, and the temperature distribution of the pipeline is assumed to be consistent, it is possible to use the following equation to calculate the temperature distribution over time [9].

$$\frac{T(t) - T_\infty}{T_i - T_\infty} = e^{-bt} \tag{32}$$

$$b = \frac{hA}{\rho V C_p} \tag{33}$$

Here, $T(t)$ is the temperature of the pipeline over time, T_∞ is the inlet temperature of the helium, h the convection heat transfer coefficient, A the contact area of the pipeline where heat transfer takes place, ρ the density of the fluid, and V and C_p the volume and specific heat of the fluid, respectively. Actually, the temperature distribution of the pipeline is not consistent as the fluid flows, but if the flux is small, it is possible to use the Biot number to verify the adequacy of the same temperature distribution. The following table shows the temperature distribution of the pipeline over time and the Biot number when the velocity of the helium is 5 m/s.

As shown in the above table it takes about 607 seconds for the temperature of the pipeline to drop to 200K, and the Biot number calculated until this point is smaller than 0.1. Therefore, it is possible to verify the adequacy of the lumped system analysis. The temperature graph of the inflowing helium over time is illustrated in Fig. 3.

If the inflowing fluid is helium, the pressure wave speed, density, internal pressure and coefficient of friction will be expressed as a function of time. Accordingly, it must be calculated over time, and included in governing Eqs. (28)-(31).

For turbulent flow the coefficient of friction inside a smooth pipe can be based on the following equation proposed by Blasius.

Table 1. Cooling time & temperature of the pipe.

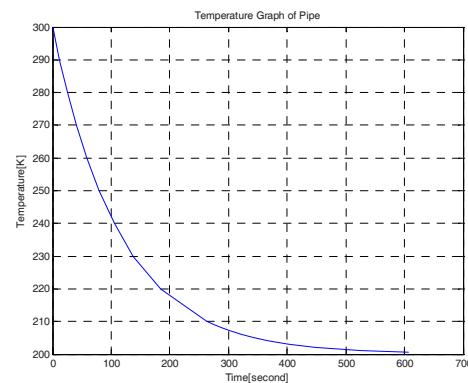| Biot Number	Time(second)	Temperature(K)
0	0	300
0.0383	12.07	290
0.0393	25	280
0.0402	40.86	270
0.0412	58	260
0.0422	79.45	250
0.0432	105	240
0.0444	138	230
0.0455	184	220
0.0467	263	210
0.0480	276	209
0.0481	289.5	208
0.0482	304	207
0.0483	322	206
0.0485	343	205
0.0486	368	204
0.0487	401.9	203
0.0488	448	202
0.0489	527	201
0.0491	607	200.5

Fig. 3. Cooling time graph of the pipe conveying cryogenic helium.

$$f = \frac{0.316}{(N_{Re})^{1/4}} \tag{34}$$

The convection heat transfer coefficient is a function of the Nusselt number and Prandtl number; the following equation proposed by Dittus-Boelter can be used to calculate the Nusselt number [10].

$$Nu = 0.023 N_{Re}^{0.8} Pr^n \tag{35}$$

Here, N_u is the Nusselt number, and exponent n is 0.4 if heat is absorbed, and 0.3 in case of cooling. To

analyze the fluid-induced vibration of the above system, the equation related to the heat transfer proposed above must be used to assign the values calculated above to each time step for analysis.

2.4 Influence on compressibility

When a cryogenic fluid flows inside a pipe of normal temperature, the temperature of the fluid increases, whereas the temperature of the pipe decreases. If the fluid is a compressible gas in this case, the density changes as the temperature changes, and the compressibility effect occurs as a result. Accordingly, it must be taken into consideration. The compressible flow can be largely divided into the flow accompanied by heat transfer (Rayleigh flow) and the flow accompanied by friction (Fanno flow) for analysis. Here, the influence of the flow accompanied by heat transfer is dominant, and as the coefficient of friction of a gas is smaller than that of a liquid, the influence of the flow accompanied by friction will be ignored for analysis. The Rayleigh analysis method was used to analyze the variation of the velocity, pressure and density of the inflowing helium over time. To this end the energy equation and the equation of ideal gas state were used, and as helium behaves very much like an ideal gas, no separate compressible factor was considered. The changes of the velocity, pressure and density of the inflowing helium are illustrated in Fig. 4.
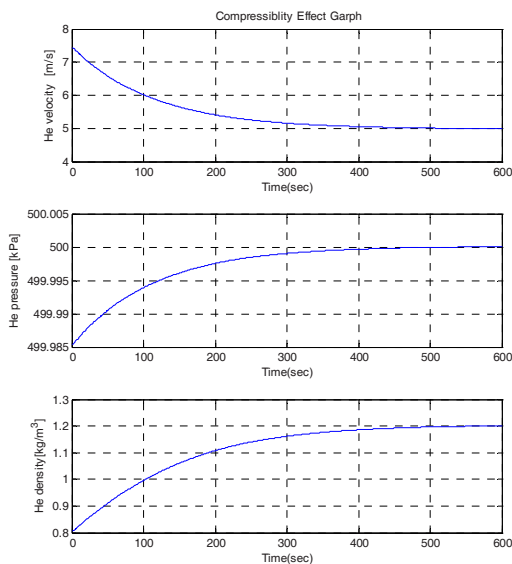

Fig. 4. Velocity, pressure and density graph of helium considering compressibility effect.

As illustrated in Fig. 4, it is clear that the pressure does not deviate greatly from the initially assumed value over time. However, the density of helium decreases in relation to the temperature, and even if the fluid flows in at a consistent velocity, it changes due to the internal heat transfer effect. As the density and velocity of the fluid are important parameters in analyzing fluid induced vibration, this compressibility effect must also be taken into consideration.

3. Numerical analysis

3.1 Numerical example

To prove the adequacy of the simulation result analyzed by means of Eqs. (28)~(31), the examples published in existing papers were used for comparison with the analysis result of this paper. [3] Since the fluid-induced vibration of the pipe has never been analyzed in consideration of the cooling effect until now, it was compared with the result of the calculation in the normal temperature range, and the adequacy of the created program was verified. The pipe is straight and its thickness is consistent, and it is simply supported as illustrated in Fig. 2. Its length is 6m, inside diameter is 32.12 mm, and thickness is 1.4mm. The modulus of elasticity of the pipe is 117 GPa, and the mass per unit length of the pipe is m_p = 1.318 kg/m. The fluid flowing inside is water, and the mass per unit length is m_w = 0.8 kg/m, and the tension initially starting at both ends of the pipe is 82 N. First, to analyze the stability of the pipe, the natural frequency was calculated. As the natural frequency of the pipe is expressed as a function of the mass and elasticity of the governing equation, depending on how the solution is assumed, a number of high-order vibrations are expressed. Here, the 1st, 2nd and 3rd natural frequencies were calculated. As the natural frequency changes in relation to the velocity, the velocity is assumed to be 0 m/s for calculation [11].

Table 2 shows the y direction natural frequencies calculated as the actually created program and the

Table 2. Comparisons of natural frequencies.

Frequency (Hz)	This method	SEM	FEM
1st	1.47	1.47	1.47
2nd	5.89	5.89	5.89
3rd	13.26	13.26	13.26

spectral element analysis method were used, and as the finite element method was used. As shown in the table, the calculation results were the same in all 3 cases. Accordingly, it was possible to verify the adequacy of this program to a certain extent.

Next, divergence and flutter were analyzed. Fig. 5 illustrates the y direction response curve of the pipeline at the flux where divergence between 0 and 2.5 seconds can take place, and flutter can occur.

Divergence occurs when the flux of the fluid makes the first natural frequency 0, while flutter occurs when the flux makes the second natural frequency 0. Divergence indicates that the response of the pipeline continues to increase without vibration, and flutter means that the amplitude of the vibration abruptly increases over time. The flux at which divergence can take place by means of the above program is 28.659 m/s, whereas the flux at which flutter can occur is 57.33 m/s, and this value was exactly the same as the value proposed in previous papers.
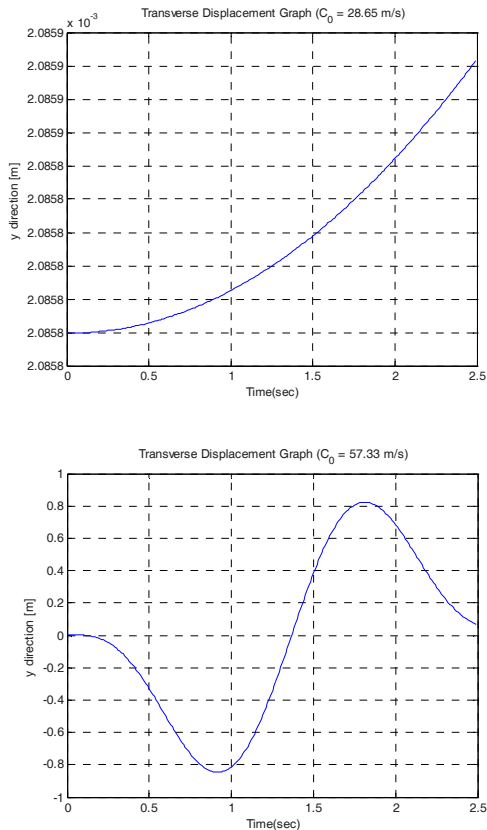

Fig. 5. Transverse displacement of the pipeline at a divergence and a flutter velocity.

The upper response curve in Fig. 5 is divergence, while the lower response curve is flutter. This trend is similar to what was presented in previous papers, and there can be slight differences depending on initial conditions. It was possible to verify the adequacy of the program based on the method derived here by means of the above result.

3.2 Numerical analysis of actual system

3.2.1 Stability analysis

To conduct analysis in consideration of the cooling effect of an actual system, the stability of the system was analyzed first. For quantitative comparison of the influence of the cooling effect, the results with consideration of the cooling effect and without were compared. The velocity at which unstable divergence occurs at normal temperature is 5662 m/s, and the flux is big because the mass per unit length of helium is very small. Accordingly, as in an actual system, the operating range of the flux is far smaller than the above velocity; unstable divergence and flutter are not very meaningful.

If fluid flows in a cryogenic pipeline, even when the flux is consistent, the temperature of the pipeline changes over time. Accordingly, the natural frequency of the pipeline changes as well. As the temperature of the pipeline changes, the modulus of elasticity of the pipeline changes and the flux changes too because of the compressibility effect. Fig. 6 illustrates the modulus of elasticity graph relative to the temperature of the pipeline.

If the flux is consistent, the natural frequency does not change at a normal temperature. Accordingly, the changes in natural frequencies over time in consideration of the cooling effect were simulated. Fig. 7 illustrates the variation of the 1st and 2nd natural frequency relative to the y direction of the pipeline.
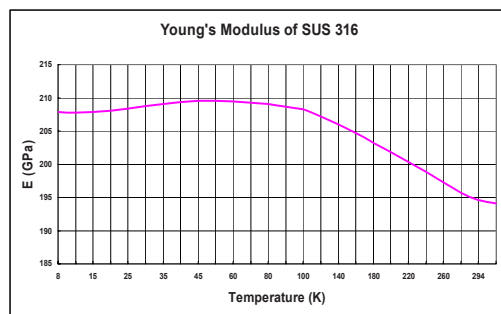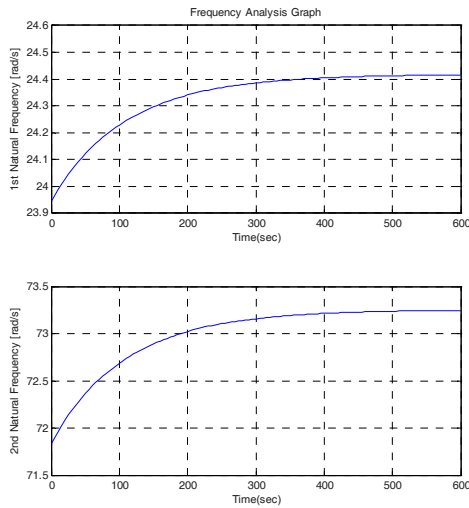

Fig. 6. Young's modulus graph relative to temperature.

Fig. 7. 1st and 2nd natural frequency of y direction.

Fig. 8. *y* direction, velocity and acceleration graphs without cooling effect.

As illustrated in the above figure, it was possible to verify the changes of the natural frequency over time even if the flux was consistent. In this case the natural frequency increases because the modulus of elasticity of the pipeline increases as the temperature drops.

3.2.2 Fluid-induced vibration analysis

As for the previously mentioned system, when the inlet flux of helium was maintained constant at 5 m/s, the y direction displacement, velocity and acceleration of the center of the pipeline were analyzed. As for the initial conditions of the pipeline, 0.05 m of displacement from the center was applied in the y direction at first, and the case with the cooling effect and that without were analyzed, respectively.

Fig. 8 illustrates the y direction displacement, velocity, acceleration graphs in case helium of the normal temperature without consideration of the cooling effect flows into the pipeline. If the flux is too small, the effect of the non-linear term gets smaller. The coupling between the actual y direction motion and the lateral motion may not be greatly affected. If the above analysis result is reviewed, the vibration does not decrease over time, and continues instead. As shown in Eq. (27), the force term is not included in the equation of y direction motion. Accordingly, it is similar to the equation of motion of free vibration, and in this case the damping matrix exists in the form of a skew-symmetric matrix. Therefore, if there is flux, vibration continues without any decrease.

Fig. 9 illustrates the displacement, velocity and acceleration graphs in consideration of the cooling
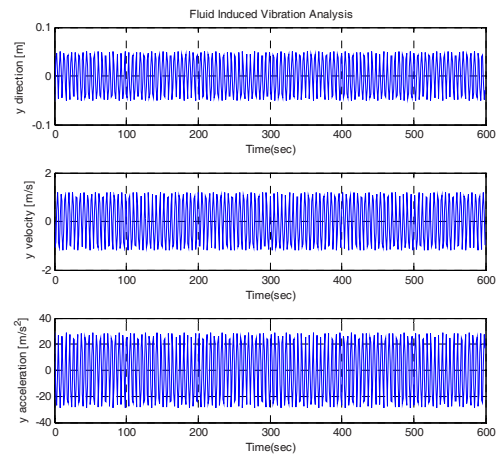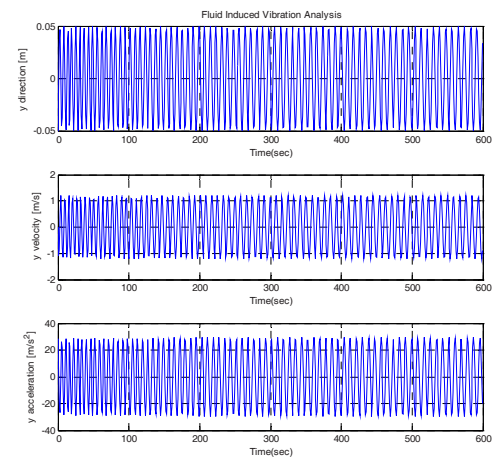

Fig. 9. *y* direction, velocity and acceleration graphs by considering a cooling effect.

effect. Comparing Fig. 8 and Fig. 9, as the number of vibrations changes in Fig. 7, Fig. 9 also shows the vibration characteristics according to the changes in the number of vibrations.

This can be attributed to the fact that the cooling effect increased the modulus of elasticity of the pipeline (Fig. 6), and thus changed the number of vibrations over time. As the modulus of elasticity of the pipeline continued to change with time, it was difficult to analyze it over a certain period of time, and so the analysis was done for the entire time period. As a result, the effect is not clearly shown.

In the difference between the analysis results of the two graphs, it will be easy to see the difference in the actual vibration, and it was illustrated in graphs.
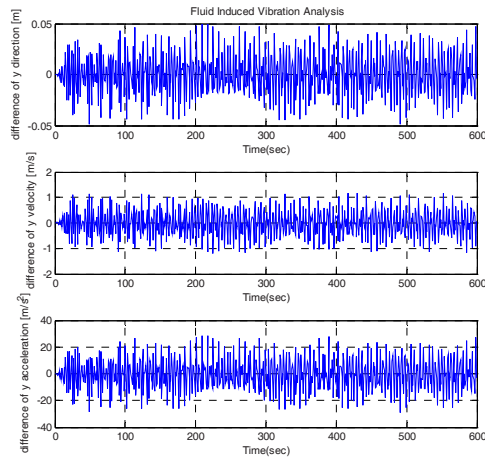
Fig. 10. Absolute difference graph of the *y* direction, velocity and acceleration between a cooling effect and a non cooling effect.
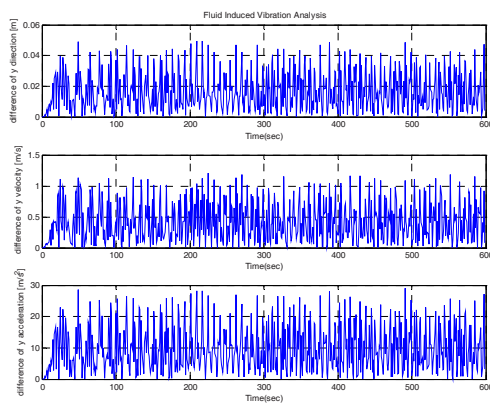

Fig. 11. Absolute value graph of the *y* direction, velocity and acceleration between a cooling effect and a non cooling effect.

Fig. 10 graphs were obtained by subtracting the cooling effect from the analysis value at the normal temperature. As clearly shown in the figure, the result at the normal temperature and that considering the cooling effect are not the same. If the cooling effect did not affect the vibration of the pipeline, Fig. 10 would show 0. In addition, the analysis was carried out when the difference in temperature was 100K. If the temperature difference is greater, it can be predicted that the difference in vibration will be greater as well. Also, as the property of the pipeline greatly affected by the cooling effect of the pipeline is the modulus of elasticity, the vibration of the pipeline may be different depending on whether the size of the modulus of elasticity increases or decreases in relation to the temperature.

Fig. 11 illustrates the graphs obtained by subtracting the absolute value of the result at the normal temperature and the result of taking the cooling effect. Likewise, the above graphs clearly show the difference between the case where the cooling effect is considered and the case where the cooling effect is not considered. This difference is thought to be caused by the fact that the natural frequency of the system at the normal temperature is different from that in consideration of the cooling effect. It would be necessary to take the cooling effect into consideration for analysis if the modulus of elasticity of the pipeline greatly changed in relation to the temperature, or there was a big difference in temperature.

4. Conclusions

In this paper the fluid induced vibration in consideration of the cooling effect of the pipeline according to the inflow of the fluid was analyzed. Unlike the analysis at the normal temperature, the cooling effect changes the properties of the fluid and the pipeline, and accordingly, the natural frequency changes as well, thereby changing the shape of the vibration.

This study proposed a method of analyzing this problem. If the inflow of a cryogenic fluid changes the physical properties of the pipeline, the most prevalent physical property is the modulus of elasticity. Therefore, data on the modulus of elasticity changing in accordance with temperature is extracted, and the Biot Number is used to check if it is possible to analyze this system by means of the lumped method. If the lumped method proves to be valid afterwards, it will be possible to calculate the temperature distribution of the pipeline according time. It will be possible if the compressibility effect of the fluid according to temperature is taken into consideration, and the influence of the changing velocity and pressure of the fluid are reflected in the equation, and the fluid-induced vibration is analyzed over a given amount of time. The example given in this study assumed that the cryogenic helium maintains a temperature of 200K, and the initial velocity of the fluid, i.e., 5 m/s, remains constant. As the initial temperature of the pipeline was assumed to be 300K, and the inlet temperature of the fluid was 200K and it took about 600 seconds to cool down the pipeline, the transverse displacement, velocity and acceleration until this moment were calculated.

For analysis of the actual responses and the ampli-

tude, the analysis time must be short, but since the time it takes about 600 seconds before the temperature of the entire pipeline becomes the same as the inlet temperature of helium in this case, it was difficult to conduct the simulation for a relatively long period of time for confirmation of the accurate difference of the two responses. Accordingly, the difference in responses was confirmed in a graph. It was possible to confirm that the size of the response varied depending on whether the cooling effect was considered or not. The reason is thought to be that the properties of the pipeline change in relation to the temperature, and consequently the natural frequency of the pipeline changes as well. If the difference between the inlet temperature of the fluid and that of the pipeline is big, or if the modulus of elasticity of the pipeline responds sensitively to the temperature, fluid-induced vibration needs to be analyzed in advance in consideration of the cooling effect. Finally, it seems possible to use this analysis method to analyze fluid-induced vibration in relation to materials with varying temperatures.

Acknowledgement

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